A Distributed Particle Filter for Bearings-Only Tracking on Spherical Surfaces

Jun Ye Yu, Mark J. Coates, Michael G. Rabbat, and Stephane Blouin

***Abstract*—We present a distributed particle filter for bearings-only tracking of a target moving on the surface of a sphere, such as Earth. The proposed filter accounts for the curvature of the surface in the measurement model for more robust performance. In addition, a linearization of the likelihood function significantly reduces the communication overhead. Simulations demonstrate that the proposed distributed approach maintains accuracy comparable to that of a centralized filter with access to all measurements even when the sensors and target are spread over a large region, where a planar approximation would fail.**

***Index Terms*—Bearings-only tracking, distributed particle filter, gossip algorithms, likelihood approximation.**

I. Introduction

WE PRESENT a novel distributed particle filter for bearings-only tracking of a single target, building on the *constraint sufficient statistics distributed particle filter* (CSSDPF) [1], [2]. Although the Earth's surface may be approximated as flat over small regions, this approximation is poor when the tracking region is large. Disregarding the Earth's curvature introduces non-negligible errors in bearing estimates, degrading tracking performance. The proposed filter incorporates the curvature directly in the measurement model, rather than using a planar approximation. Simulations illustrate that it provides tracking accuracy comparable to that of a centralized particle filter regardless of the distances between the target and sensors while enjoying a low communication overhead. The proposed filter may be particularly of interest for applications involving underwater acoustic signals, which may propagate and be sensed over thousands of kilometers [3].

A. Related Work

Distributed particle filters have recently attracted attention because they are less susceptible to sensor failure [4]. Each sensor calculates the particle weights based on its individual measurements and all sensors then gossip to reach a consensus [5], [6] on the particle weights. This can be achieved by either transmitting particle weights or sufficient statistics (e.g., [4], [7]–[9]). However, reaching a consensus on all individual particle weights involves considerable communication overhead. Algorithms seeking to reduce this overhead focus communication resources on a subset of most likely particles [10], [11], or extract a set of sufficient statistics to directly approximate the likelihood function [1], [2], [12].

II. Bearings Equations

Let (x_1, x_2) and (s_1, s_2) denote the longitude and latitude of a target and sensor, respectively, on the surface of a sphere. Under the approximation that the sphere has zero eccentricity, the bearing angle from the target to the sensor is [13]

$$\operatorname{atan2}\left(\frac{\sin(x_1 - s_1)\cos(x_2)}{\cos(s_2)\sin(x_2) - \sin(s_2)\cos(x_2)\cos(x_1 - s_1)}\right), \tag{1}$$

where $\operatorname{atan2}(\cdot)$ denotes the four-quadrant inverse tangent. Using the small angle approximations $\sin(\theta) \approx \theta$ and $\cos(\theta) \approx 1$ for $\theta \approx 0$, along with standard trigonometric identities, if $x_1 \approx s_1$ and $x_2 \approx s_2$ then (1) simplifies to

$$\operatorname{atan2}\left(\frac{x_1 - s_1}{x_2 - s_2}\cos(x_2)\right). \tag{2}$$

If (x_1, x_2) and (s_1, s_2) are instead treated as coordinates in the two-dimensional Cartesian plane then the bearing angle from the target to the sensor is

$$\operatorname{atan2}\left(\frac{x_1 - s_1}{x_2 - s_2}\right). \tag{3}$$

Compared with (2), this is equivalent up to the factor of $\cos(x_2)$, which accounts for how latitude and longitude warp near the poles (i.e., as x_2 approaches $\pm\pi/2$ radians). The planar approximation, using (3) in place of (1), is thus justified when the target is close to the sensor and the target is near the equator ($x_2 \approx 0$). When the target is not near the equator, the planar approximation is still applicable as long as the small-angle approximation is valid, after an appropriate transformation of the coordinates; i.e., by projecting onto the plane tangent to the sphere at the target's position [14].

III. Distributed Bearings-Only Particle Filters

The CSSDPF [1], [2] is a powerful approach to distributed particle filtering for single-target bearings-only tracking under an additive white Gaussian noise measurement model. The CSSDPF simplifies the distributed posterior update calculations by noticing that, under the bearings-plus-Gaussian noise

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measurement model, the posterior can be approximated using six statistics. Rather than fusing posterior log-likelihood values for individual particles, the posterior update only requires fusing these statistics, which can be accomplished with significantly less communication. Then, the likelihood of any particle can be evaluated as a function of the six statistics.

Mohammadi and Asif [1] derive expressions for the sufficient statistics under the planar bearing model (3). In this section we derive equations for sufficient statistics directly using the spherical bearings model (1). We consider a network of K sensors with positions (longitude/latitude) $s^k = (s_1^k, s_2^k)$, $k = 1, \ldots, K$, which we assume to be known and fixed. To implement a distributed particle filter [4], at each time step the network must communicate to evaluate the joint likelihood given the observations at all sensors. Let us focus on a particular time step and omit any notational dependence on time. Let z^k denote the measurement at sensor k, and for $i = 1, \ldots, N$ let $x^i = (x_1^i, x_2^i)$ denote the ith particle position. We wish to compute the joint likelihoods $p(z^1, \ldots, z^K | x^i)$ for all particles in a communication-efficient manner.

B. Measurement Model and Transformation

For a sensor at position (s_1^k, s_2^k), we write the observed noisy bearing angle to a target at position (x_1, x_2) as

$$z^k = \operatorname{atan2}(L_1^k / L_2^k) + \zeta^k, \tag{4}$$

where [cf. equation (1)]

$$L_1^k = \sin(x_1 - s_1^k)\cos(x_2) \tag{5}$$

$$L_2^k = \cos(s_2^k)\sin(x_2) - \sin(s_2^k)\cos(x_2)\cos(x_1 - s_1^k), \tag{6}$$

and ζ^k is zero-mean wrapped normal noise with parameter σ.

We make the standard assumption that the noise terms ζ^k at different sensors are statistically independent. Then the observations are conditionally independent given the target position $x = (x_1, x_2)$, and so the joint likelihood factorizes:

$$p(z^1, \ldots, z^K | x) = \prod_{k=1}^{K} p(z^k | x). \tag{7}$$

By rearranging (4) and using the definition of the tangent function, we find that, given the sensor and target positions, the observation z^k and noise ζ^k are equivalently related as

$$\sin(2z^k)L_2^k - \cos(2z^k)L_1^k = \sin(2\zeta^k)L_2^k + \cos(2\zeta^k)L_1^k. \tag{8}$$

Let $\tilde{z}^k \stackrel{\text{def}}{=} \sin(2z^k)L_2^k - \cos(2z^k)L_1^k$. Taking the first-order Taylor approximation of the right-hand side at $\zeta^k = 0$ gives

$$\sin(2\zeta^k)L_2^k + \cos(2\zeta^k)L_1^k \approx 2\zeta^k L_2^k + L_1^k + o(|\zeta|^2),$$

which suggests that we can approximate $\tilde{z}^k$ as following a Gaussian distribution when ζ is sufficiently small (equivalently, when σ^2 is sufficiently small).

B. Gaussian Likelihood Approximation

Adopting the approximation that $\tilde{z}^k$ follows a Gaussian distribution, we derive expressions for its mean and variance. From Euler's formula and the characteristic function of a Gaussian random variable, we obtain

$$\mathbb{E}[\sin(2\zeta^k)L_2^k + \cos(2\zeta^k)L_1^k | x] = e^{-2\sigma^2} L_1^k. \tag{9}$$

Let $R_k(x)$ denote the variance of $\sin(2\zeta^k)L_2^k + \cos(2\zeta^k)L_1^k$ for a given target state $x = (x_1, x_2)$. Using standard trigonometric identities, Euler's formula, and the characteristic function of a Gaussian random variable, we obtain

$$R_k(x) = \frac{1 - e^{-8\sigma^2}}{2}(L_2^k)^2 + \frac{1 + e^{-8\sigma^2}}{2}(L_1^k)^2 - e^{-4\sigma^2}(L_1^k)^2. \tag{10}$$

Using the Gaussian approximation, the log-likelihood of a particle x^i given the measurement at sensor k is

$$\log p(z^k | x^i) \approx -\frac{\left(\sin(2z^k)L_2^k - \cos(2z^k)L_1^k - e^{-2\sigma^2}L_1^k\right)^2}{2R_k(x^i)} - \frac{1}{2}\log\left(2\pi R_k(x^i)\right), \tag{11}$$

where we use x_1^i and x_2^i in place of x_1 and x_2 when computing L_1^k and L_2^k; see eqns. (5) and (6).

C. Expansion and Sufficient Statistics

Let $\beta_k = \cos(2z^k) + e^{-2\sigma^2}$ and consider the first term in (11). By expanding the square, using standard trigonometric identities, and rearranging terms, we are able to write

$$\frac{\left(\sin(2z^k)^E L_2^k - \cos(2z^k)L_1^k - L_1^k e^{-2\sigma^2}\right)^2}{2R_k(x^i)} = \sum_{j=1}^{6} \frac{G_j(x^i)C_j^k}{2R_k(x^i)} \tag{12}$$

where

$$\begin{aligned}
C_1^k &= \sin^2(2z^k)\sin^2(s_1^k)\sin^2(s_2^k) + \beta_k^2\cos^2(s_1^k)\\
&\quad + 2\beta_k\sin(2z^k)\sin(s_1^k)\cos(s_1^k)\sin(s_2^k)\\
C_2^k &= \sin^2(2z^k)\cos^2(s_1^k)\sin^2(s_2^k) + \beta_k^2\sin^2(s_1^k)\\
&\quad - 2\beta_k\sin(2z^k)\sin(s_1^k)\cos(s_1^k)\sin(s_2^k)\\
C_3^k &= 2\sin^2(2z^k)\sin(s_1^k)\cos(s_1^k)\sin^2(s_2^k)\\
&\quad - 2\beta_k^2\sin(s_1^k)\cos(s_1^k)\\
&\quad - 2\beta_k\sin(2z^k)\left(1 - 2\cos^2(s_1^k)\right)\sin(s_2^k)\\
C_4^k &= \sin^2(2z^k)\cos^2(s_2^k)\\
C_5^k &= -2\sin^2(2z^k)\cos(s_1^k)\sin(s_2^k)\cos(s_2^k)\\
&\quad + 2\beta_k\sin(2z^k)\sin(s_1^k)\cos(s_2^k)\\
C_6^k &= -2\sin^2(2z^k)\sin(s_1^k)\sin(s_2^k)\cos(s_2^k)\\
&\quad - 2\beta_k\sin(2z^k)\cos(s_1^k)\cos(s_2^k)\\
G_1(x^i) &= \sin^2(x_1^i)\cos^2(x_2^i)\\
G_2(x^i) &= \cos^2(x_1^i)\cos^2(x_2^i)\\
G_3(x^i) &= \sin(x_1^i)\cos(x_1^i)\cos^2(x_2^i)\\
G_4(x^i) &= \sin^2(x_2^i)\\
G_5(x^i) &= \cos(x_1^i)\sin(x_2^i)\cos(x_2^i)\\
G_6(x^i) &= \sin(x_1^i)\sin(x_2^i)\cos(x_2^i).
\end{aligned}$$

Note that the terms $G_j(x^i)$ only depend on the particle coordinates and not on any quantities specific to sensor k. Similarly, the terms C_j^k only depend on information specific to sensor k and not on the particle coordinates.

Using (12) in (11), we approximate the joint log-likelihood of particle x^i as

$$\log p(z^1,\ldots,z^K|x^i) \approx -\sum_{j=1}^{6} G_j(x^i)\sum_{k=1}^{K}\frac{C_j^k}{2R_k(x^i)} - \frac{1}{2}\sum_{k=1}^{K}\log\left(2\pi R_k(x^i)\right). \tag{13}$$

D. Variance Approximation

The local factors, $\frac{C_j^k}{2R_k(x^i)}$, depend on the particle through the denominator $R_k(x^i)$. To use (13) in a distributed particle filter, one must compute and communicate the factors individually for each particle. With N particles, this involves communicating $6N$ values, a significant overhead. To eliminate the dependence on the number of particles, we replace the per-particle variance $R_k(x^i)$ of (10) with the weighted average,

$$\widehat{R}_k = \sum_{i=1}^{N} w_k^i R_k(x^i), \tag{14}$$

where w_k^i is the weight that node k assigns to particle x^i. Substituting $\widehat{R}_k$ for $R_k(x^i)$ in (13) yields our final expression for the approximate log-likelihood,

$$\log p(z^1,\ldots,z^K|x^i) \approx -\sum_{j=1}^{6} G_j(x^i)\sum_{k=1}^{K}\frac{C_j^k}{2\widehat{R}_k} - \frac{1}{2}\sum_{k=1}^{K}\log\left(2\pi \widehat{R}_k\right). \tag{15}$$

With this approximation, fusing the measurements from all sensors simplifies to computing the six statistics $\sum_{k=1}^{K}\frac{C_j^k}{2\widehat{R}_k}$ for $j=1,\ldots,6$. Each statistic is a sum of values at each node, which is straightforward to compute in a distributed manner using gossip algorithms [6]. With the six statistics in hand, any node can compute the joint likelihood of any particle x^i. In particular, the communication overhead no longer depends on the number of particles being used. Note that the normalization term $-\frac{1}{2}\sum_{k=1}^{K}\log(2\pi\widehat{R}_k)$ can be dropped since it is identical for all particles.

The proposed algorithm has the same communication overhead as the CSSDPF [2] which also computes six statistics to characterize the likelihood function and uses the same variance approximation. The derivation above assumes that each sensor receives one measurement per time step. If a sensor receives multiple measurements, it calculates the local factors for each measurement and adds them to get aggregate local factors. If a sensor receives no measurement, it sets its local factors to zero. If no measurement is available at any sensor, then all global factors are zero and all particles have the same likelihood.

IV. Performance Evaluation

We evaluate and compare the performance of our proposed filter with that of two other algorithms: the CSS distributed particle filter [2] using a planar approximation, and a centralized bootstrap particle filter. To illustrate the impact of the size of the sensing region, we assess performance using simulated data. The simulated scenario is shown in Fig. 1. A single target moves in a clockwise, approximately circular trajectory over a span of 50 minutes. Three sensors, approximately equidistant from the target, obtain noisy bearings measurements once per minute, for a total of 50 time steps. We vary the inter-sensor distance from 100 km to 1000 km to illustrate its impact on methods using a planar approximation. Each sensor makes a variable number of measurements per time step, with an average of 5 measurements being obtained in total across the entire network at each time step. With a small network and few measurements per time step, it may be more efficient to flood the measurements throughout the network, but this requires all nodes to know each others' measurement functions. The choice is discussed in more detail in [4].

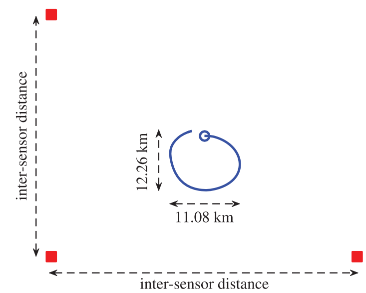

Fig. 1. The simulated scenario. The target starts at the position indicated with a circle and moves along a clock-wise trajectory over a period of 50 minutes. Three sensors, indicated by squares, are roughly equidistant from the target and the inter-sensor distance is varied between 100 km and 1000 km.

The simulations are repeated for three different starting locations for the target. The northernmost location is in the Greenland Sea (73.893°N, 0.598°W), at roughly the same latitude as Daneborg, Greenland. The next location, in the North Atlantic Ocean (42.377°N, 36.107°W) halfway between Europe and North America, is at roughly the same latitude as Boston, USA. The last location is in the South Pacific Ocean (0.238°S, 125.579°W), just south of the equator.

The simulated target dynamics switch randomly between two different motion models [15]. At any time, the target is assumed to travel at constant velocity ($m=1$) or make clockwise/counter-clockwise coordinated turn ($m=2$) with probabilities P_{cv} and $1-P_{cv}$ respectively. The target state is defined as a vector of target coordinates and velocities, $x_t=[x_{1,t},x_{2,t},\dot{x}_{1,t},\dot{x}_{2,t}]^T$, and the state at time step $t+1$ is

$$x_{t+1} = F_t^m x_t + \varepsilon_t, \tag{16}$$

where ε_t is zero-mean process noise. For constant velocity steps, the dynamic matrix F_t^m is equal to

$$F_t^1 = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{17}$$

For coordinated turn steps the dynamic matrix F_t^m is equal to

$$F_t^2 = \begin{bmatrix} 1 & 0 & \frac{\sin(\Omega_t T)}{\Omega_t} & -\frac{1-\cos(\Omega_t T)}{\Omega_t} \\ 0 & 1 & \frac{1-\cos(\Omega_t T)}{\Omega_t} & \frac{\sin(\Omega_t T)}{\Omega_t} \\ 0 & 0 & \cos(\Omega_t T) & -\sin(\Omega_t T) \\ 0 & 0 & \sin(\Omega_t T) & \cos(\Omega_t T) \end{bmatrix}, \tag{18}$$

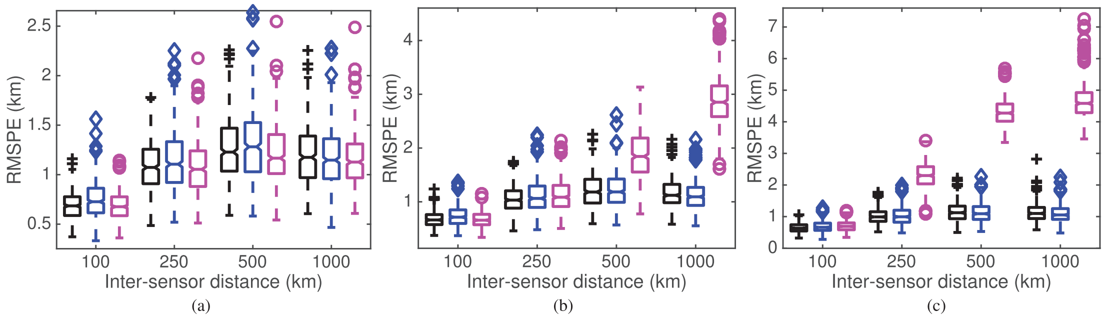

Fig. 2. Time-averaged root mean squared position error for the three algorithms compared while varying the inter-sensor distance, and with the target location at the (a) South Pacific Ocean, (b) North Atlantic Ocean, and (c) Greenland Sea. Each group of three boxplots represents the accuracy, over 500 Monte Carlo trials, of the three algorithms: (from left-to-right) centralized BPF (+), $CSS_{\text{spherical}}$ ($\diamond$), and CSS_{planar} ($\circ$).

where T is the sampling interval and Ω_t is the turning rate

$$\Omega_t = \frac{a}{\sqrt{\dot{x}_{1,t}^2 + \dot{x}_{2,t}^2}}, \tag{19}$$

with $a \in \mathbb{R}$ being the manoeuvre acceleration parameter ($a < 0$ corresponding to a clockwise turn). The process noise $\varepsilon_t \in \mathbb{R}^4$ is implemented in the plane tangent to the sphere at the target location. The distance traveled per time step is very small, so this approximation is extremely accurate, and the resulting noise is zero-mean Gaussian with covariance matrix [16]

$$Q = \sigma_a^2 \begin{bmatrix} \frac{T^3}{3} & 0 & \frac{T^2}{2} & 0 \\ 0 & \frac{T^3}{3} & 0 & \frac{T^2}{2} \\ \frac{T^2}{2} & 0 & T & 0 \\ 0 & \frac{T^2}{2} & 0 & T \end{bmatrix}. \tag{20}$$

The parameter settings used in the simulations are $P_{cv} = 0.1$, $a = -0.001$, $\sigma_a = 10^{-4}$, and $T = 1$ minute. Measurements are generated based on (1), and there is zero-mean additive Gaussian measurement noise with variance $\sigma^2 = 0.0076$ rad^2, equivalent to a standard deviation of 5 degrees.

The proposed distributed filter implements a bootstrap particle filter at each node, and the likelihood evaluations are approximated using (15) with the statistics computed using gossip protocols (abbreviated $CSS_{\text{spherical}}$). We compare with the performance of the CSSDPF [2] derived using the planar bearing equation (CSS_{planar}) where the sensor and particle coordinates are projected onto the plane tangent to the Earth's surface at the posterior mean estimate of the target position at the previous time step before evaluating the likelihood. The likelihood evaluations of both distributed particle filters involve approximation. We also compare their performance with that of a centralized *bootstrap particle filter* (BPF) which has access to all sensor measurements and uses exact likelihood evaluations based on the spherical bearing equation (1) with additive Gaussian noise. All filters use 1000 particles, and we assume that the local filters in the distributed algorithms remain exactly synchronized from iteration-to-iteration by using a shared source of randomness and max consensus iterations.

All three particle filters use the same dynamic model, and they only differ in the likelihood evaluation. At each time step, particles are resampled and disturbed with zero-mean Gaussian noise with variance $\sigma_p^2 = 10^{-6}$ for regularization.

We measure accuracy using the time-averaged *root mean squared position error* (RMSPE), averaged over 500 Monte Carlo trials for each simulation scenario. Distances are measured along geodesics on the Earth's surface. The relative sensor positions and target trajectory are the same for all trials. However, the process noise realization, measurement noise realization, and the source of randomness used by the particle filter differ from trial to trial.

Fig. 2 compares the RMSPE for the three algorithms while varying the inter-sensor distance and the starting position. Near the equator (S. Pacific Ocean), the three methods give comparable performance over distances of up to 1000 km. However, in the N. Atlantic Ocean scenario, when the inter-sensor distance is at least 500 km there is noticeable degradation in the performance of CSS_{planar}, and this effect is stronger at latitudes closer to the poles. The accuracy of $CSS_{\text{spherical}}$ remains consistently comparable to that of the centralized BPF regardless of the inter-sensor distance and the latitude.

Additional simulations illustrate that evaluating the CSS log-likelihood with the variance approximation (15) instead of using per-particle variances, as in (13), both reduces the communication overhead and has a regularizing effect, leading to more accurate log-likelihoods; see the supplemental material.

V. Conclusion

We have presented a new distributed particle filter for bearings-only tracking that incorporates the curvature of the sensing region in the measurement model for more accurate performance. We make two approximations (that a transformation of the observations is Gaussian distributed, and that the variance can be approximated using the average variance over all particles) to significantly reduce the communication overhead. The resulting filter only requires computing six statistics over the network, and so the communication overhead is independent of the number of particles used. Tests using simulated data show that accounting for the curvature leads to a significant performance improvement.

We approximated the Earth's surface as a sphere. The bearing equation (1), derived in [13], models the Earth's surface more generally as an oblate spheroid. It should be possible to extend the methodology developed in this paper for tracking on the surface of an oblate spheroid or a general ellipsoid. The likelihood approximation applies when tracking a single target over a spherical surface. An interesting avenue for future work is the extension to the multiple-target setting.

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