Bayesian Reasoning with Constructive Neural Networks

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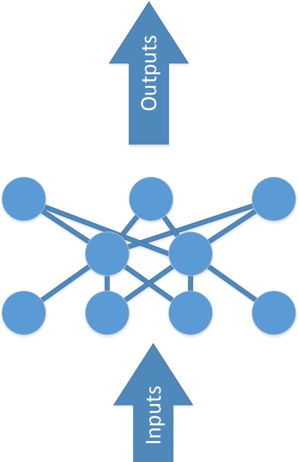
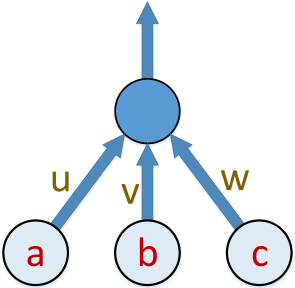
Neural Networks in Brief

- Network of interconnected units and weights
- Conceptually like simplified neurons.
- Goal: learn associations between inputs and outputs (and generalize)



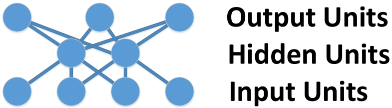
Neural Networks in Brief

- Units are simple mathematical functions:
  - **Input**: weighted sum of outputs of other units,
    $ua + vb + wc$
  - **Output**: a nonlinear function of input
    $f(ua + vb + wc)$
  - Send output to other units doing the same
- Modify weights to reduce the error

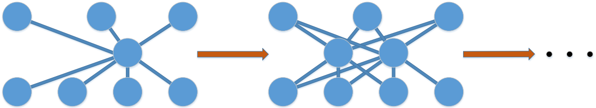


Static VS Constructive Networks

- Static networks:
  - Network's structure fixed in advance (heuristically)
  - Only weights are learned
  - Not psychologically plausible



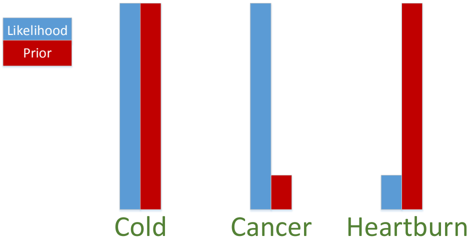
- Constructive networks:
  - Both weights and network's structure are learned
  - Similar to humans' developmental, autonomous learning



- Assume Alice coughs. Most probably, she has ...
  - Cold
  - Heartburn
  - Lung cancer

- Cough a symptom of cold and lung cancer and not heartburn
  $\rightarrow$ Likelihood: observation expectation
- Cold much more common than cancer
  $\rightarrow$ Prior: belief before observation

- Cold high in both



Bayesian Reasoning

- A set of hypotheses to choose from with some priors (e.g., cold, cancer, etc.)
- Degree of belief: Probability, a number between 0 and 1
- Observation (e.g., Alice coughing) $\rightarrow$ likelihood
- Combining prior and likelihood info:
  Bayes' rule: $\textit{posterior belief} \propto \textit{likelihood} \times \textit{prior}$
- Making decision

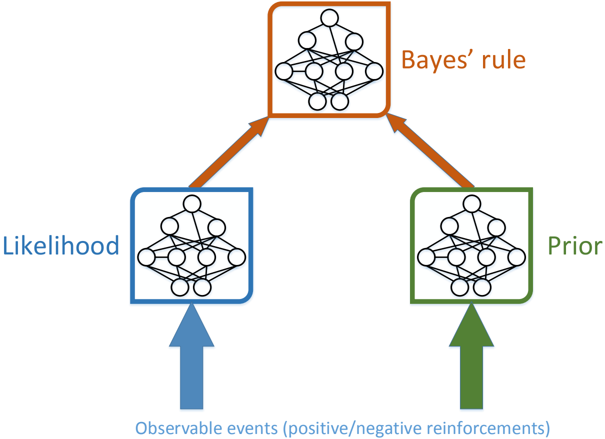
Why use neural networks?

- Challenges with Bayesian approach:
  - Only at Marr's computational level
  - Can be under–constrained
  - Deviation from optimal Bayes' rule (e.g., base–rate neglect)

- Neural nets can help with resolving these issues:
  - At implementation level & psychologically plausible
  - Likelihoods/priors learned from observable events
  - Explaining both Bayesian models and deviations

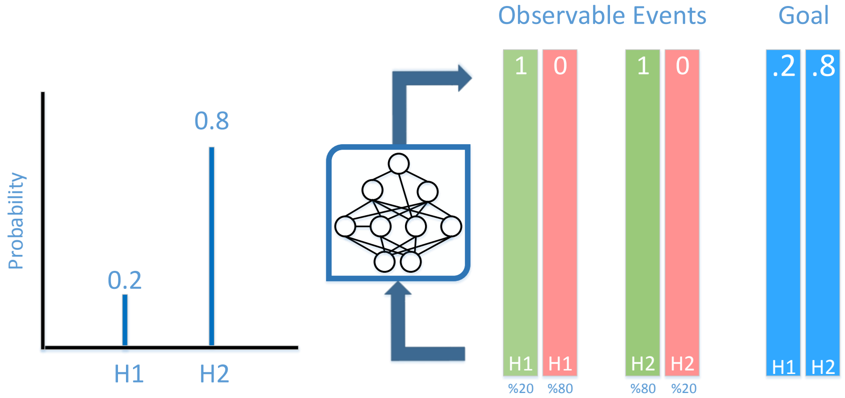
Constructive Neural Net Modeling Bayesian Reasoning

- A constructive neural network formed of three modules:
  - Representing priors
  - Representing likelihoods
  - Applying Bayes' rule



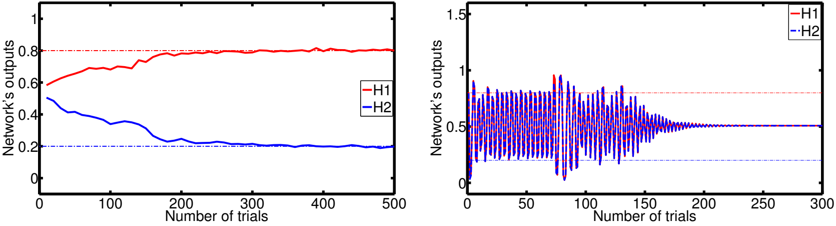
Probability Matching

- First two modules perform **Probability Matching**:
  - **Input:** hypotheses
  - **Output:** degrees of belief (probability)
  - NO access to actual probabilities
  - Only positively (1) or negatively (0) reinforced instances



- H1: $1, 0, 0, 0, 1, 0, 0, 0, \ldots \rightarrow 0.2$
- H2: $0, 1, 1, 1, 0, 1, 1, 1, \ldots \rightarrow 0.8$

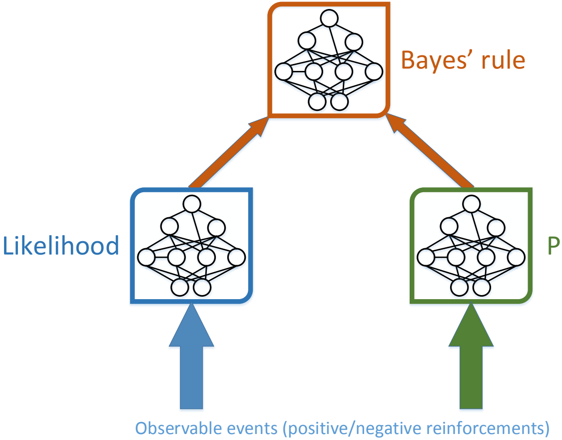
- Can neural nets successfully learn the probabilities?
  — Static networks: Not always
  — Constructive networks: Yes



(a) Constructive Neural Net (b) Static Neural Net

Bayes' Rule Module

- **Input**: likelihoods/priors from probability matching module
- Applying Bayes' rule
- **Output**: posterior beliefs



Base–rate Neglect

- Not taking full account of priors
- Basing decision only on likelihoods (assuming equal priors)

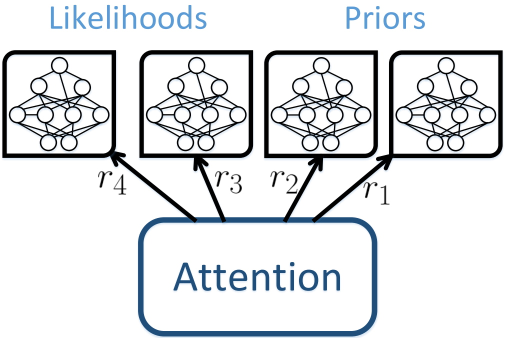
- **Example**: Tom is an opera buff who enjoys touring art museums when on vacation. Which situation is more likely?
  - Tom plays trumpet for a major symphony orchestra.
  - Tom is a farmer.

- **Reasons**: deliberate neglect, failure to recall, long–term synaptic decay, etc.

Modeling Base–rate Neglect as Weight Disruption

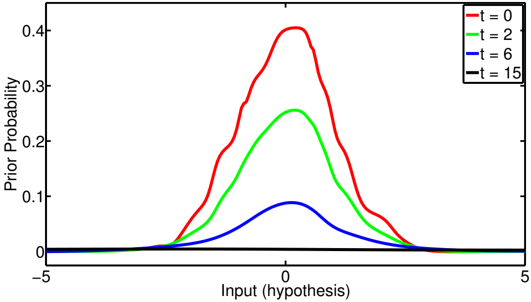
- **Attention module** applying **weight disruption**
- $w_{\text{new}} = r \times w_{\text{old}}, \quad 0 \leq r \leq 1$
- Likelihoods → $r = 1$
- Priors → $0 \leq r < 1$

- Effects of attention, memory indexing, and relevance
- Long–term synaptic decay



Base–rate Neglect Results

- $r = 0.8^t$, i.e., higher t → more weight disruption
- More disrupted weights → more equal priors → more base–rate neglect
- **Conclusion**: base-rate neglect (a deviation) can be explained in the same framework as Bayes' rule



Conclusion

- Neural net model of Bayesian Reasoning
- Realistic inputs
- Autonomous learning through constructive neural net
- Representing probabilities in neural circuitry
- Unifying Bayesian accounts AND deviations

- Bayesian models and neural nets can be viewed as being at different and complementary levels

Questions?

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