Moment based filters for multi-target tracking using super-positional sensors

Santosh Nannuru

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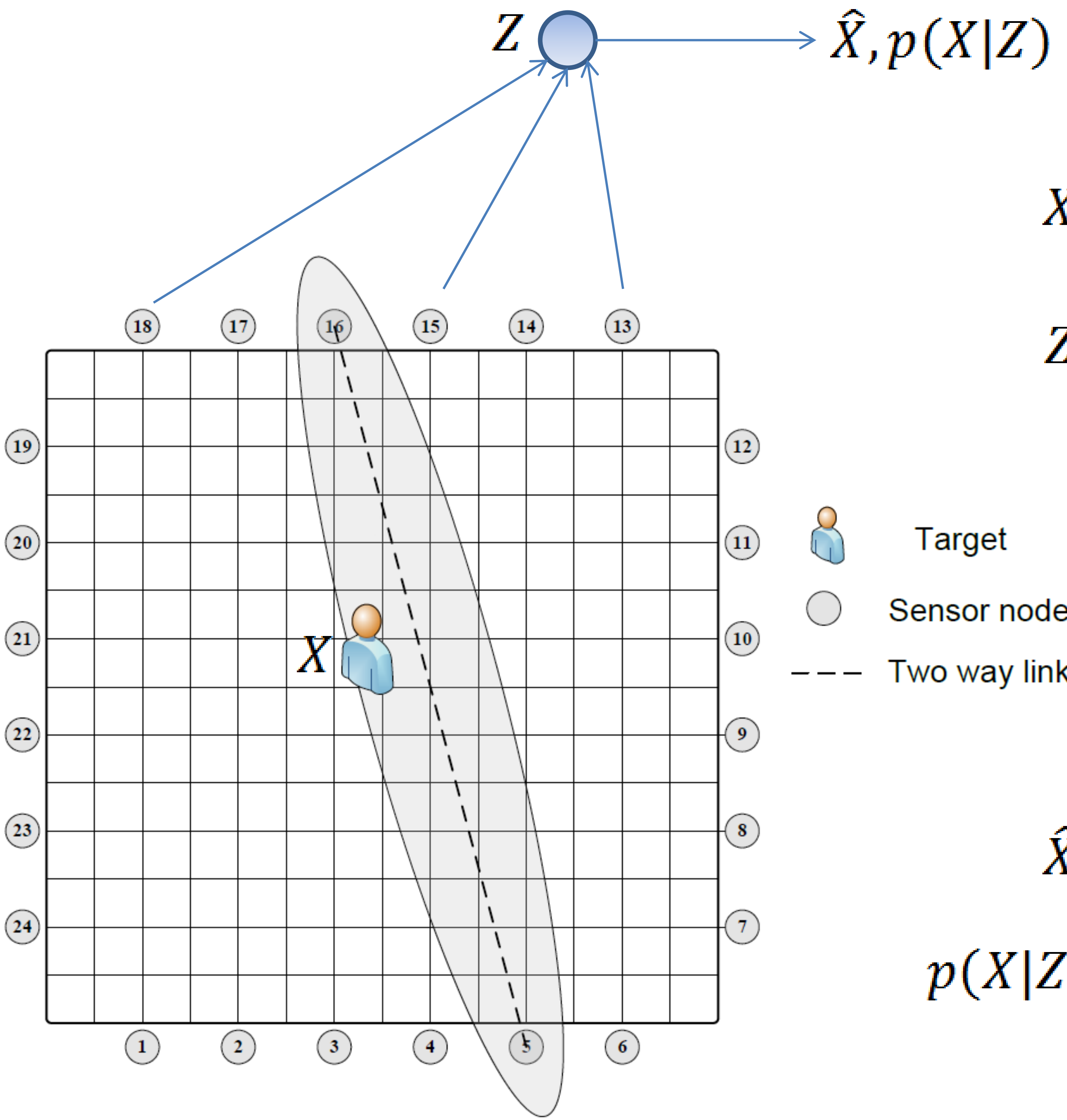


X - State

Z - Observation

$$Z = g(X)$$

$\hat{X}$ - State estimate

$p(X|Z)$ - State distribution

Outline

- Super-positional sensors
- Problem statement
- Moment filters
  - PHD filter (ALM)
  - CPHD filter
- Simulations
- Future work

Sensors

Standard sensors

- Each target produces one or no measurement

- Each measurement produced by single target or clutter

Super-positional sensors

- Targets cause additive measurement

$$z^1, z^2, z^3 \ldots z^{N_k}$$

$$z = z^1 + z^2 + \cdots + z^{N_k}$$

- Measurements are not independent

Problem – Multi-target tracking

- N_k targets; state $X_k = \{x_{1,k}, x_{2,k} \ldots x_{N_k,k}\}$
- Independent target dynamics

$$f(x_{n,k+1} | x_{n,k})$$

Example $x_{n,k+1} = x_{n,k} + w_{n,k+1}$

- Super-positional sensor observations

$$z_k^n = g(x_{n,k}) \qquad Z_k = \sum_{n=1}^{N_k} g(x_{n,k}) + W_k$$

Problem – Multi-target tracking

- Given $p(X_0)$ and observations

$$Z_{1:k} = \{Z_1, Z_2 \dots Z_k\}$$

  find state estimate $\hat{X}_k$

- Estimate state posterior $p(X_k | Z_{1:k})$

(Bayes) Optimal solution

- Bayes recursive solution
  - Prediction

$$p(X_k|Z_{1:k-1}) = \int f(X_k|X) * p(X|Z_{1:k-1})\mu(\delta X)$$

  - Update

$$p(X_k|Z_{1:k}) = \frac{g(Z_k|X_k) * p(X_k|Z_{1:k-1})}{\int g(Z_k|X) * p(X|Z_{1:k-1})\mu(\delta X)}$$

- Issues – Set integrals, no closed form solution, computationally intractable

(Bayes) Optimal solution

- Traditional solutions
  - Fixed number of targets
  - Linear and Gaussian assumption
  - Particle filters

- More recently
  - First moment based filters
  - PHD and CPHD [Mahler]
  - ALM [Thouin *et al.*]

First moment

- First moment, $E = ? \int X * p(X) * \mu(\delta X)$
- First-order multi-target moment OR

  Probability Hypothesis Density (PHD)

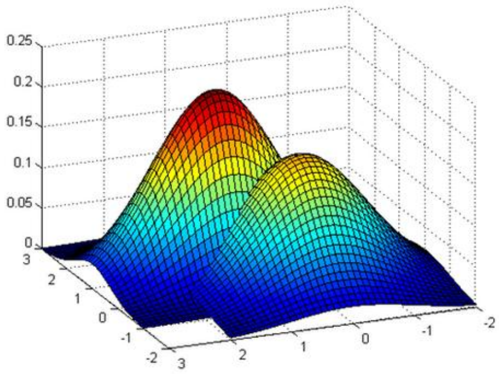
$$D(x) = \int p(\{x\} \cup X) * \mu(\delta X)$$

$$p_{k|k} \xrightarrow{prediction} p_{k+1|k} \xrightarrow{update} p_{k+1|k+1}$$

$$D_{k|k} \xrightarrow{prediction} D_{k+1|k} \xrightarrow{update} D_{k+1|k+1}$$

Properties of PHD

- Defined over single target state space
- Integration $\int D_{k|k}(x)dx = E(N_k)$
- High where targets present
- Amenable to particle methods



PHD filter (ALM)

$$D_{k|k} \xrightarrow{prediction} D_{k+1|k} \xrightarrow{update} D_{k+1|k+1}$$

- PHD prediction

$$D_{k+1|k}(x) = b_{k+1|k}(x) + \int p_S(y) * f_{k+1|k}(x|y) * D_{k|k}(y)dy$$

- PHD update

$$D_{k+1|k+1}(x) = L_{k+1}(x) * D_{k+1|k}(x)$$

- $L_{k+1}(x)$ = function(model, Z_{k+1})

PHD implementation

- Particle approximation of PHD

$$D_{k|k}(x) \approx \sum_{i=1}^{N_p} w_i * \delta(x^i)$$

- Propagate particles, update weights
- Issues
  - Estimating target number
  - Clustering particles

Cardinalized PHD (CPHD) filter

- Additionally propagate cardinality distribution

$$p^n_{k|k} \xrightarrow{prediction} p^n_{k+1|k} \xrightarrow{update} p^n_{k+1|k+1}$$

- CPHD prediction

$$p^n_{k+1|k}(m) = \sum_{j=0}^{m} p_b(m-j) * \sum_{l=j}^{\infty} C^l_j * {p_s}^j (1-p_s)^{l-j} * p^n_{k|k}(l)$$

CPHD filter

- CPHD update

$$p^n_{k+1|k+1}(m) \propto \ell_{k+1}(m) * p^n_{k+1|k}(m)$$

$$D_{k+1|k+1}(x) = L_{k+1}(x) * D_{k+1|k}(x)$$

- $\ell_{k+1}(m)$ and $L_{k+1}(x)$

  = function(model, Z_{k+1})

Particle MCMC filter

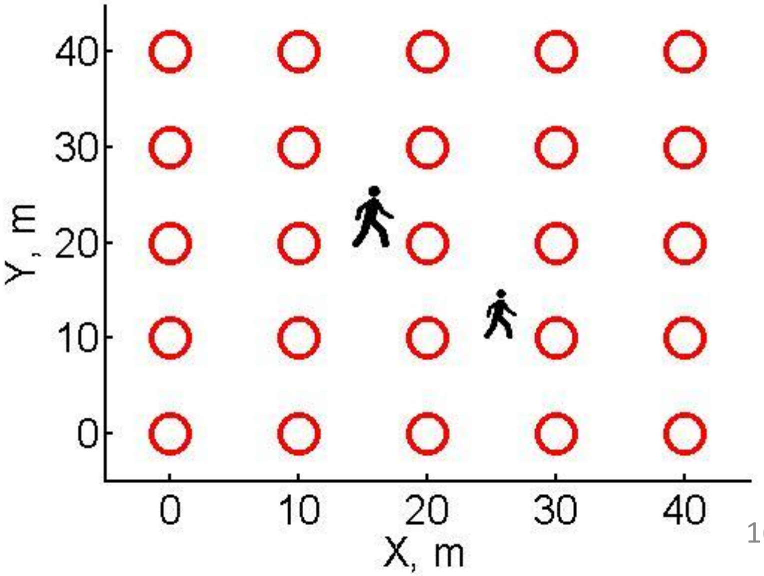
- Sample from full posterior $p(X_k|Z_{1:k})$
  - Construct a Markov Chain
  - Metropolis-Hastings sampling
  - Gibbs sampling for each target

- Handling time varying targets
  - Assumption on max. number of targets
  - Indicator variable for each target $(x_{n,k}, e_{n,k})$

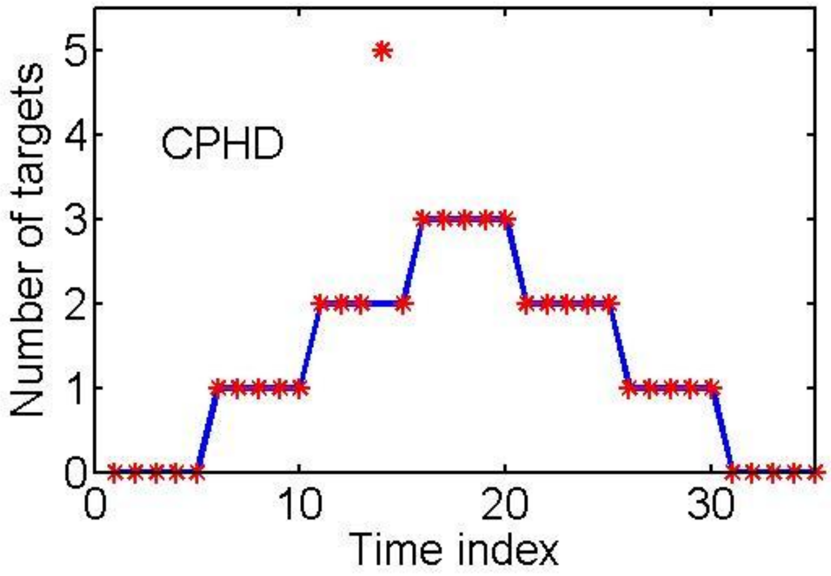
Example – Acoustic sensors

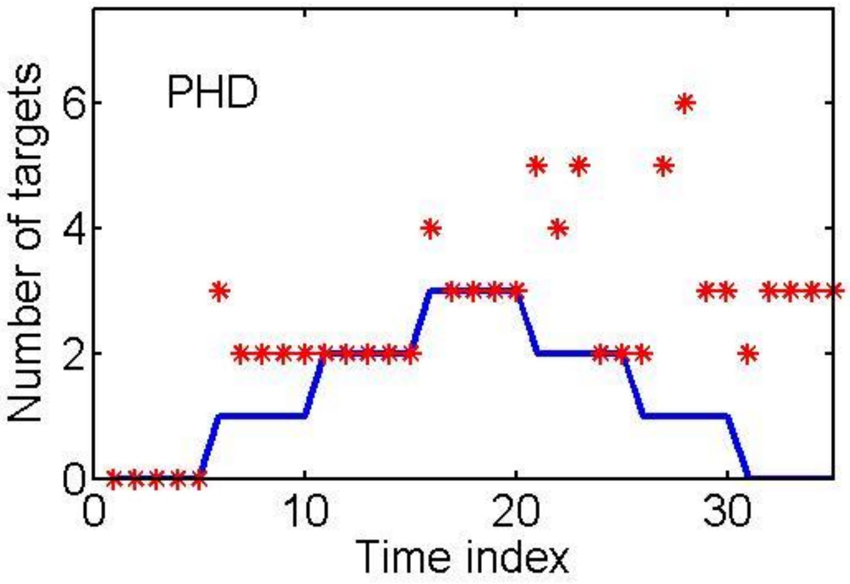
- 25 sensors deployed over 40m x 40m grid
- Active targets, communication with sensors
- Target motion – constant velocity
- Measurement vector dim – 25

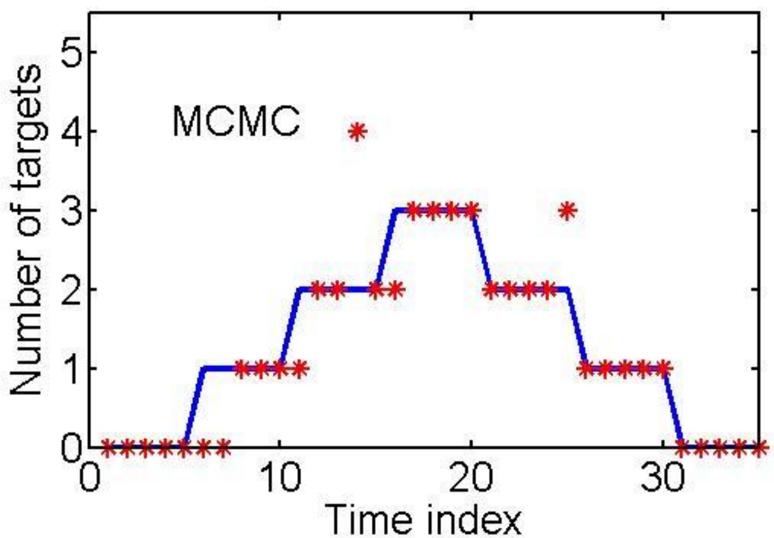
$$z = \sum_{n=1}^{N_k} \frac{A}{\max(d_0, r_n)} + w_k$$



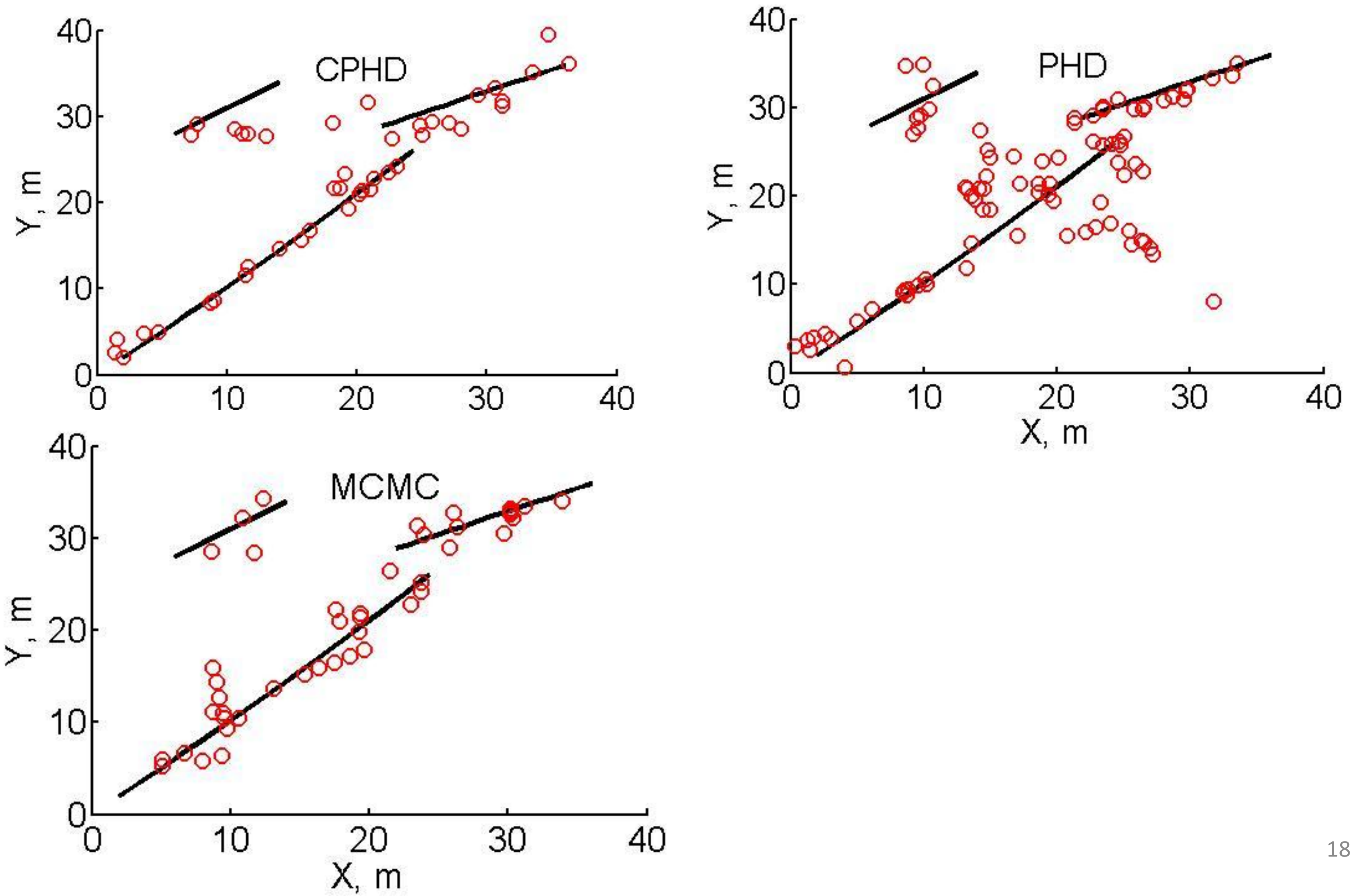
Target number estimate







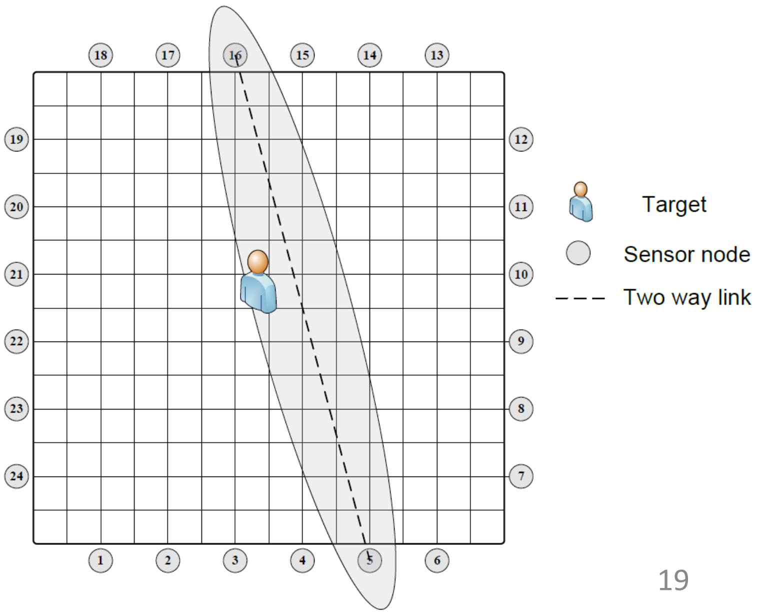
Location estimates



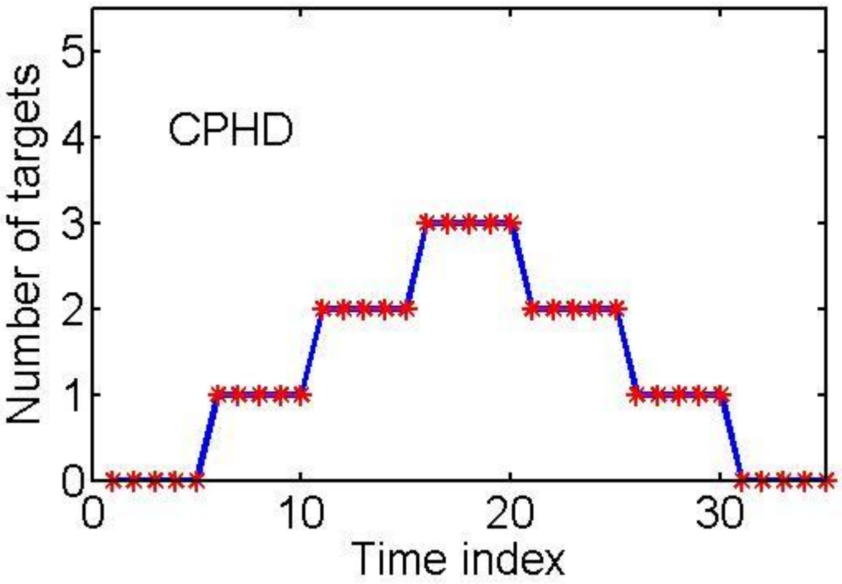
Example – RF Tomography

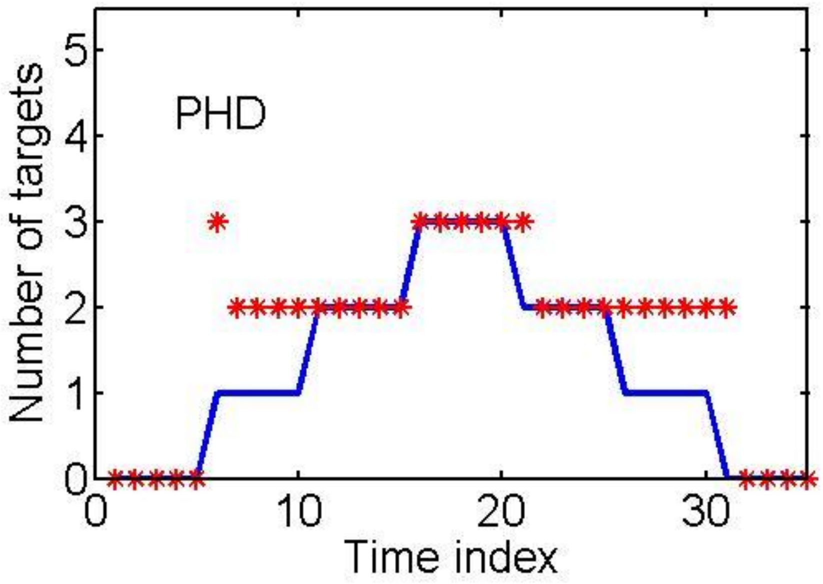
- 24 sensors deployed on periphery of 20m side square
- Passive targets, sensor pairs record RSS
- Target motion – constant velocity
- Measurement dim - 276

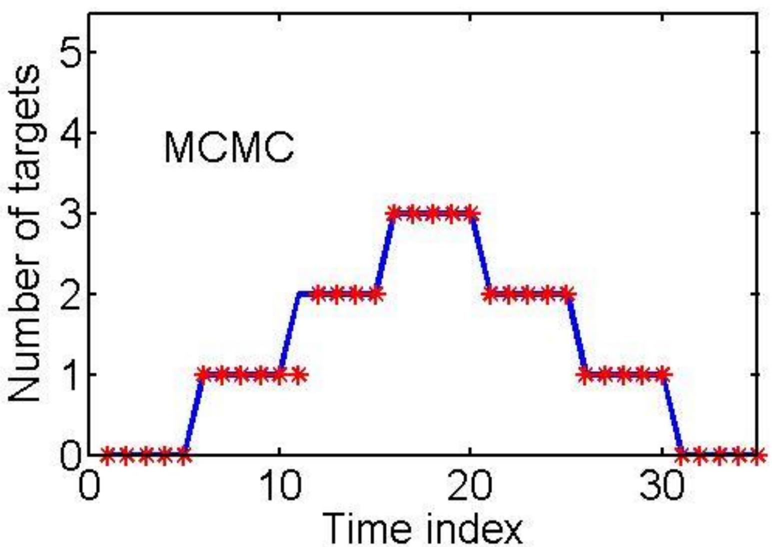
$$z = \sum_{n=1}^{N_k} \varphi * \exp\left(-\frac{\lambda_n}{\sigma_\lambda}\right) + w_k$$



Target number estimate







Computational time (s)

	CPHD	PHD	MCMC
Acoustic	41	355	354
RF Tomography	36	199	653

Issues

- Clustering of particles
  - Better clustering at every time step
  - Cluster evolution over time

- High measurement dimension - stable weight update
- Tracks - linking state estimates over time

References

[1] R. Mahler, "Multitarget Bayes filtering via first-order multitarget moments," IEEE Trans. on Aerospace and Electronic Systems, Oct. 2003.

[2] R. Mahler, "PHD filters of higher order in target number", IEEE Trans. on Aerospace and Electronic Systems, Oct. 2007.

[3] R. Mahler, "CPHD filters for superpositional sensors," in Proc. Signal and Data Processing of Small Targets, San Diego, CA, Aug. 2009.

[4] Thouin, F., Nannuru, S., and Coates, M., "Multi-target tracking for measurement models with additive contributions," Proc. 14th Int'l Conf. on Information Fusion, Chicago, July 2011.

THANK YOU !

QUESTIONS ?