Scalable MCMC in degree corrected stochastic block model

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Introduction

- Community detection from networks
- Academic collaboration, protein interaction, social networks
- Community: dense internal and sparse external connections
- Earlier approaches: hierarchical clustering, modularity optimization, spectral clustering, clique percolation
- Challenges: handling sparsity, scalability

Introduction

- Heuristic objective function, greedy optimization
- Plethora of techniques[1] [2]
- Numerous quality metrics[3]
- Principled approach: statistical modelling of community structures

[1] S. Fortunato, "Community detection in graphs," *Phys. Rep.*, vol. 486, pp. 75–174, Feb. 2010.

[2] S. Parthasarathy, Y. Ruan, and V. Satuluri, "Community discovery in social networks: Applications, methods and emerging trends," in *Social Network Data Analytics*, pp. 79–113. Springer US, Boston, MA, Mar. 2011.

[3] T. Chakraborty, A. Dalmia, A. Mukherjee, and N. Ganguly,"Metrics for community analysis: a survey, *ACM Comput. Surv.*, vol. 50, no. 4, pp. 1–37, Aug. 2017.

Stochastic Block Model (SBM)

- Connectivity depends on community membership[4].

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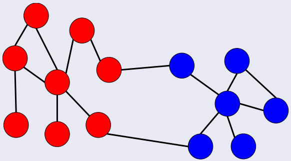

N: no. nodes, K: no. communities

c_i: membership of node i

$c_i \in \{1, 2, ..., K\}$, $\mathcal{C} = \{c_i\}_{i=1}^N$

$y_{ab} \in \{0, 1\}$: (a, b)'th entry in adj. matrix

$\beta_{k\ell} \in (0, 1)$: link probability between two nodes in community k and ℓ

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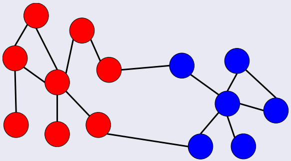

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$$y_{ab}|(c_a = k, c_b = \ell) \sim Poisson(\omega_{k\ell}),$$
$$\mathcal{L}(\mathbf{y}|\mathcal{C}, \omega) = \sum_{k,\ell}(m_{k\ell} \log \omega_{k\ell} - n_k n_\ell \omega_{k\ell}),$$

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where, $m_{k\ell} = \sum_{a,b} y_{ab} \mathbf{1}_{\{c_a = k, c_b = \ell\}}$ and $n_k = \sum_a \mathbf{1}_{\{c_a = k\}}$

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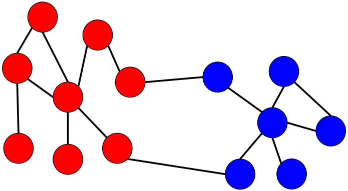
Greedy algorithm

pick a random node, place it in a community to maximally increase the objective.

Degree Corrected Stochastic Block Model (DC-SBM)



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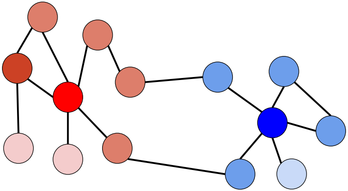

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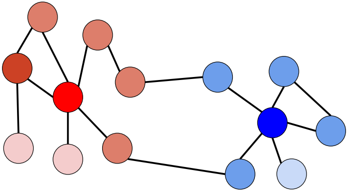

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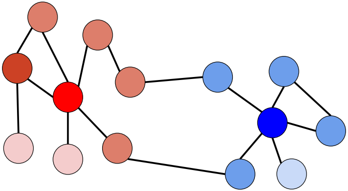

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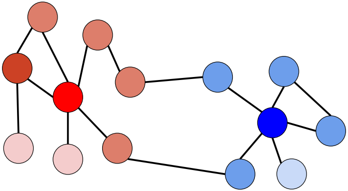

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where, $\kappa_k = \sum_a d_a \mathbf{1}_{\{c_a = k\}} \,.$

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- Overlapping communities[6]

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- Posterior inference of $p(\beta, \pi | \mathbf{y})$
- assortative MMSB (a-MMSB): $\beta_{k\ell} = \delta$ for $k \neq \ell$

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  - Stochastic gradient Riemannian Langevin dynamics (SGRLD)
  - Faster convergence
  - Better approximation of posterior

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- Preconditioning: Riemannian Langevin Dynamics (RLD)

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Stochastic gradient Langevin dynamics (SGLD)

- complexity in LD: $\mathcal{O}(N)$

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- asymptotic convergence[10] to the posterior distribution

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Numerical Experiments and Results

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Edges	2742	14996	25998	118521

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- evaluation metrics:
  - average perplexity:

$$perp_{avg}(\mathbf{Y}_{\mathbf{test}} | \{\pi^{(i)}, q^{(i)}, r^{(i)}\}_{i=1}^{T}) = \exp\left(- \frac{\sum_{y_{ab} \in \mathbf{Y}_{\mathbf{test}}} \log \left\{ \frac{1}{T} \sum_{i=1}^{T} p(y_{ab} | \pi^{(i)}, q^{(i)}, r^{(i)}) \right\}}{|\mathbf{Y}_{\mathbf{test}}|} \right).$$

  - area under ROC (AUC) for link prediction task

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Convergence of Perplexity

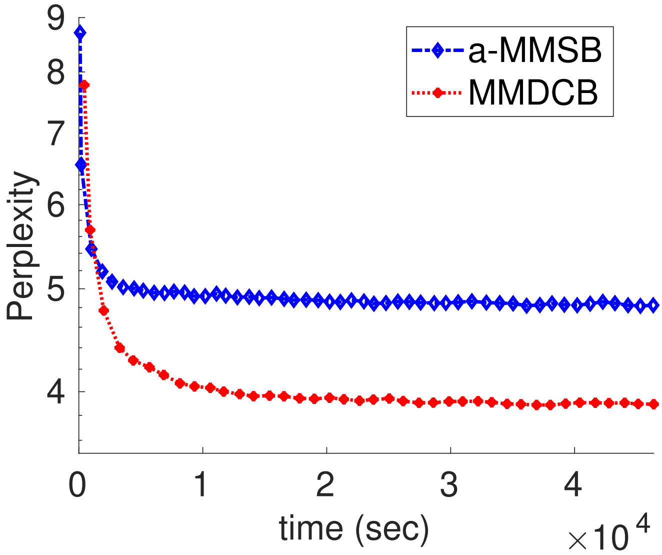

Figure: Convergence of perplexity for HEP-PH dataset, $K = 50$

Comparison of Perplexity at convergence

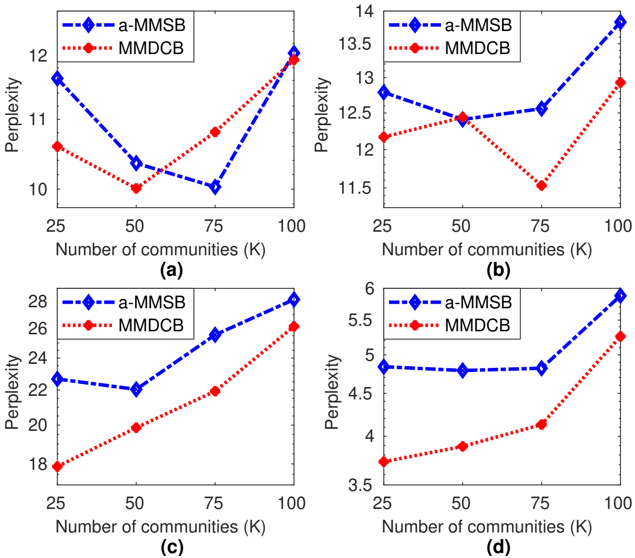

Figure: (a) NETSCIENCE, (b) RELATIVITY, (c) HEP-TH and (d) HEP-PH

Comparison of AUC at convergence

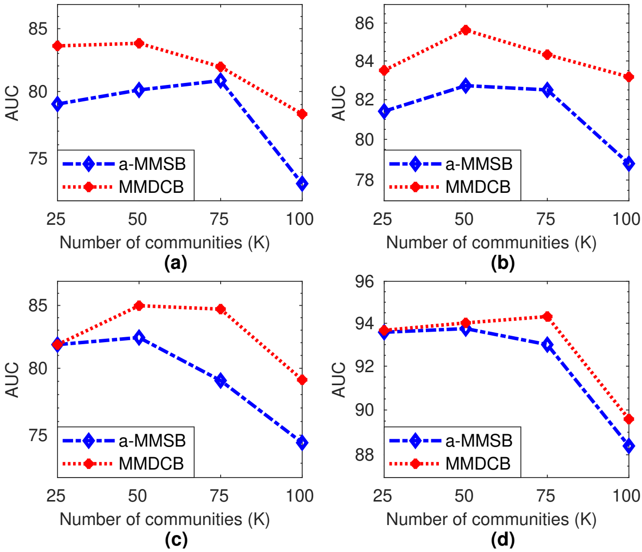

Figure: (a) NETSCIENCE, (b) RELATIVITY, (c) HEP-TH and (d) HEP-PH

Conclusion

- MMDCB models the observed graph better than a-MMSB.
- SG-MCMC algorithms scale well to large networks.
- Future work:
  - better generative models
  - efficient mini-batch sampling, variance reduction
  - more advanced SG-MCMC algorithms